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Discovery Processes and the Kondratieff Cycle: Mathematical Principles

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Abstract

'Discovery processes' are hypothesised as processes in which there is an interplay between innovation activity and exploitation activity. Both require societal resources, in a zero sum game, so that, while innovation is needed to make exploitation possible, exploitation takes effort away from innovation, inhibiting the maintenance let alone expansion of exploitation. Such negative feedbacks, which propagate through the economy with a certain lag, give rise to oscillatory behaviour. Hence economic expansion proceeds in cycles. The cycle duration is linked to the human ageing process, since it is those entering adulthood who are best placed to respond to changed economic opportunities, while older adults are more committed to existing occupations. Since innovation activity is always positive (things are seldom de-invented), the cumulative activity (total inventions) manifests as quasi-logistic pulses, where growth is flatter when society's focus has shifted towards exploitation and steeper when society's focus has shifted back towards innovation. 'Complex' discovery processes extend the logic of these 'simple' discovery processes with an additional dynamic whereby high value exploitation processes stimulate competition, which reduces their value, while innovation restores their value. The coupled cycles of innovation, exploitation and competition produce a fluctuation in all three activities, whose phase relationships correspond to those of the Kondratieff cycle and whose duration, given some assumptions about the turnover of generations, can be shown to match the Kondratieff period. The ideas discussed in the paper are applied to manned space exploration and are used to estimate the growth of the human presence in orbit over the next half-century.

Keywords: *economic cycles, long wave, logistics, mathematical modelling.*

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1. Discovery Processes

1.1. Simple Discovery Processes

In 1419, the Prince Dom Henrique of Portugal, known in the English-speaking world as Prince Henry the Navigator, was appointed governor of the Algarve. He proceeded to sponsor journeys of exploration to the Atlantic islands and down the West African coast (Findlay and O'Rourke 2007: 145–147). By Henry's death in 1460, this initiative was beginning to stall. The Portuguese had reached the Senegal and Gambia rivers and were preoccupied with the rich trading opportunities they offered (Cunliffe 2017: 520–522). It was not until 1469, when the Portuguese king commissioned a Lisbon merchant to continue the exploration effort that southward progress resumed, with Bartolemeu Dias passing the Cape of Good Hope in 1488 and Vasco da Gama reaching India ten years later.

Thus, Portuguese exploration led to the discovery of commercial opportunities, and exploitation of those opportunities diverted effort away from exploration, which slowed down exploration. Once exploitation had become saturated, or was sufficiently routine as to be transferred to private interests, royal exploration took off again.

This is an example of what will here be called a *simple discovery process* (see Fig. 1). Exploration exerted a positive effect on exploitation that is caused exploitation to increase. On the other hand, exploitation exerted negative feedback on exploration (*i.e.,* caused exploration to decrease).

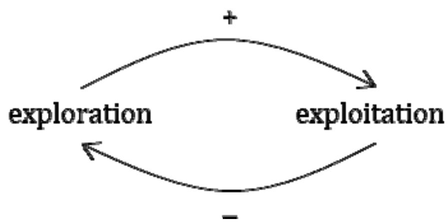


Fig. 1. A simple discovery process: exploration and exploitation

Another example of a simple discovery process is found in the activity of the European trading companies in the Americas and the Far East during early modern times. The high prices being obtained for spices and exotic goods stimulated exploration for more supplies, and the resulting increase in supplies then brought the price down, which caused exploration to fall (Phillips 1990: 50). One can note that price caused an increase in exploration and exploration caused a decrease in price.

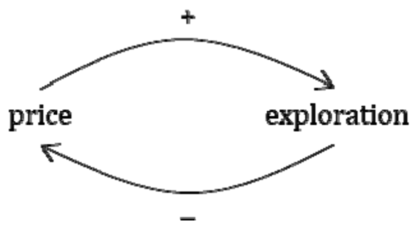


Fig. 2. A simple discovery process: price and exploration

Students of historical cycles will likely feel a sense of recognition at the 50-year interval between Prince Henry's inauguration of an exploration programme in 1419 and its renewal in 1469. It recalls the 50- to 60-year duration of the Kondratieff cycle in economic affairs (Grinin, Devezas, and Korotayev 2012). A period roughly reminiscent of the Kondratieff cycle is also found in the fluctuations in Europe's colonial trade that resulted from the dynamic illustrated in Fig. 2 (see Fig. 3 which shows sugar imports of the Dutch Vereenigde Oostindische Compagnie [VOC]).

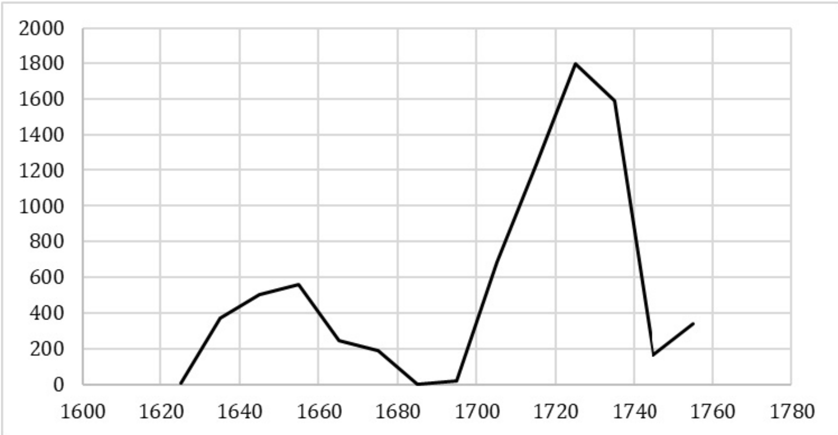


Fig. 3. VOC sugar imports from Asia (tons/year)

Source: Steensgaard 1990: 134.

Marchetti (1980, 1986) has shown that this kind of '50-year pulsation in human affairs' typically has a logistic (S-shaped) form, whereby the rate of discovery initially accelerates then slows down and tapers off (see Fig. 4, showing the wave of inventions centred on the year 1857). Marchetti has shown that the same pattern applies for earlier and later invention waves, as well as for a whole range of other socio-technical phenomena, such as the construction of metro systems. His work has been continued by Modis (1992, 2002, 2013).

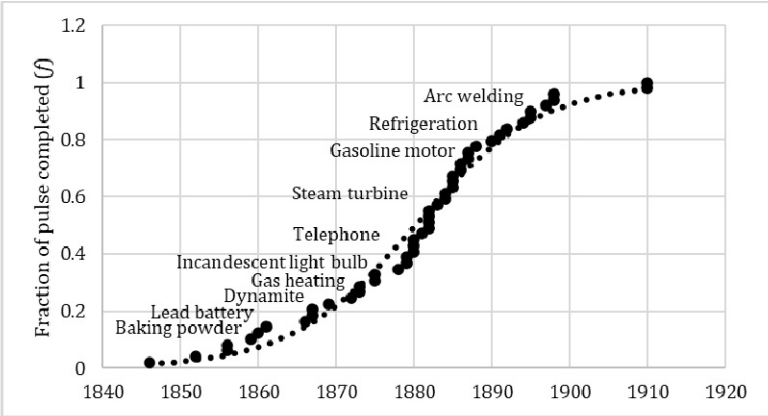


Fig. 4. Invention wave centred on 1857. The dashed line shows an idealised logistic curve fitted to the points

Source: Marchetti 1980.

Marchetti attributes the logistic behaviour of invention waves to the fact that they involve learning processes. Inventions initially spread through a receptive society, meeting existing needs, and organising industries and distribution networks around themselves. As society processes, absorbs, and incorporates these inventions, the capacity to accept further inventions decreases, leading to saturation. Once full uptake has been achieved and the economy has settled into a stable configuration, factors like population growth, fundamental discoveries, and the emergence of new needs can then trigger the next pulse.

We can understand Marchetti's invention waves as another kind of discovery process. In this case, innovation leads to implementation (*i.e.,* bringing the innovation into use), whereas increased focus on implementation reduces the effort devoted to innovation (see Fig. 5).

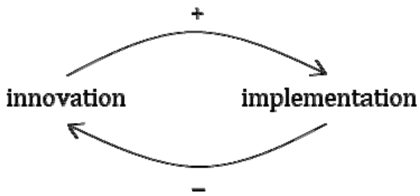


Fig. 5. A simple discovery process: innovation and implementation

Marchetti typically plots logistic pulses not in the raw form of Fig. 4 but in the transformed form $\log(f/(1-f))$ where f is the fraction of the pulse completed so far. The advantage of this is that a logistic process then appears as a straight line, allowing for a simple visual check (see Fig. 6, which replots the data of Fig. 4 in this form).

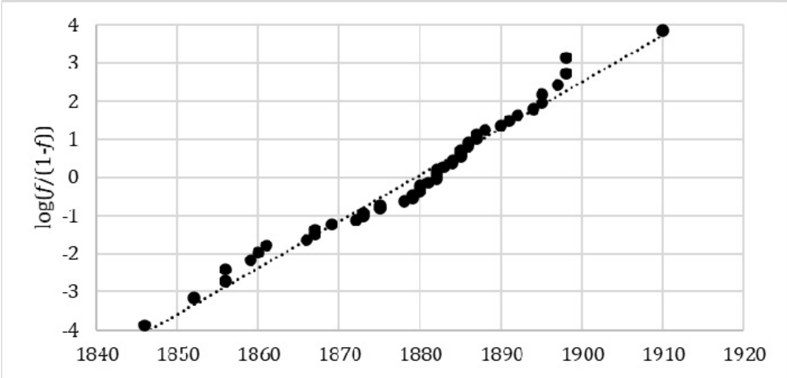


Fig. 6. Alternative presentation of data shown in Fig. 4

One can use the method of Fig. 6 to plot the progress of European exploration, from the early Portuguese voyages into the 20th century (see Fig. 7). The activity of exploration turns out to have been divided into a series of logistic pulses, where the duration of each pulse is about a century. Thus, it seems that discovery processes can operate over different timescales and do not necessarily exhibit the Kondratieff period.

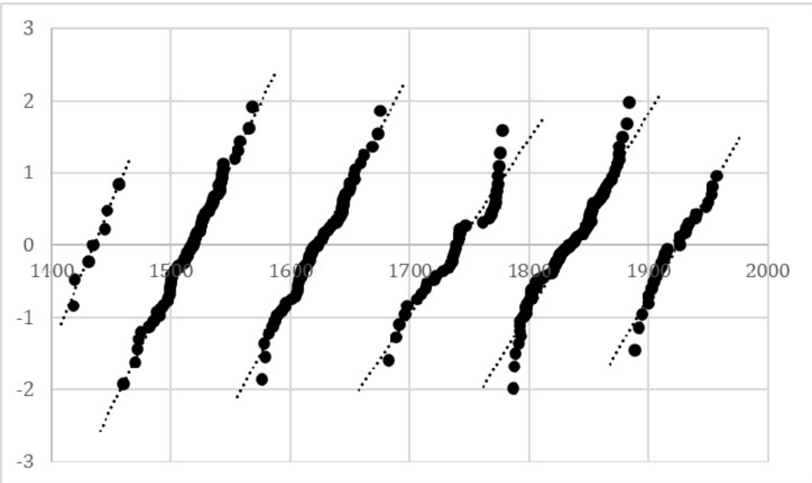


Fig. 7. Logistic pulses in the European voyages of discovery. Each point in the figure represents a discovery, and these are plotted in terms of time and log(*f*/(1 – f)) where *f* is the fraction of each pulse completed

Source: Manjunath n.d.

Two other discovery processes with non-Kondratieff timescales are the colonisation of Mediterranean islands and the colonisation of the Pacific Ocean. Fig. 8 depicts Mediterranean colonisation in the 1st Millennium BC and shows that it had a logistic form. The timescale was some 700 years. Fig. 9 depicts the approximate dates of colonisation of the Pacific islands and shows that they divide into a series of distinct groups, with 500 to 1000 years between each colonisation pulse. In this case, the dates of colonisation are not known with sufficient accuracy to show that the pulses were logistic. Nevertheless, it remains clear that some negative feedback must have applied, so that each pulse lost its momentum, and then later another pulse would begin.

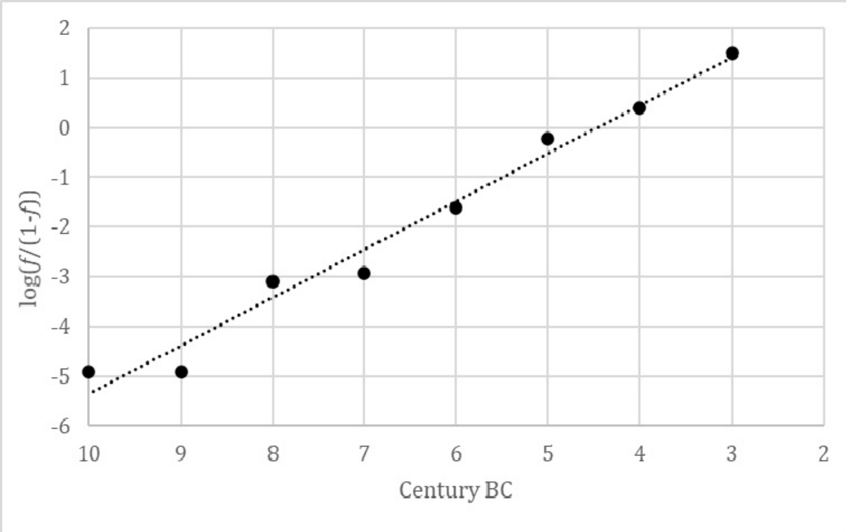


Fig. 8. Colonisation of the Mediterranean in terms of the fraction of islands discovered at each date

Source: Cherry 1981.

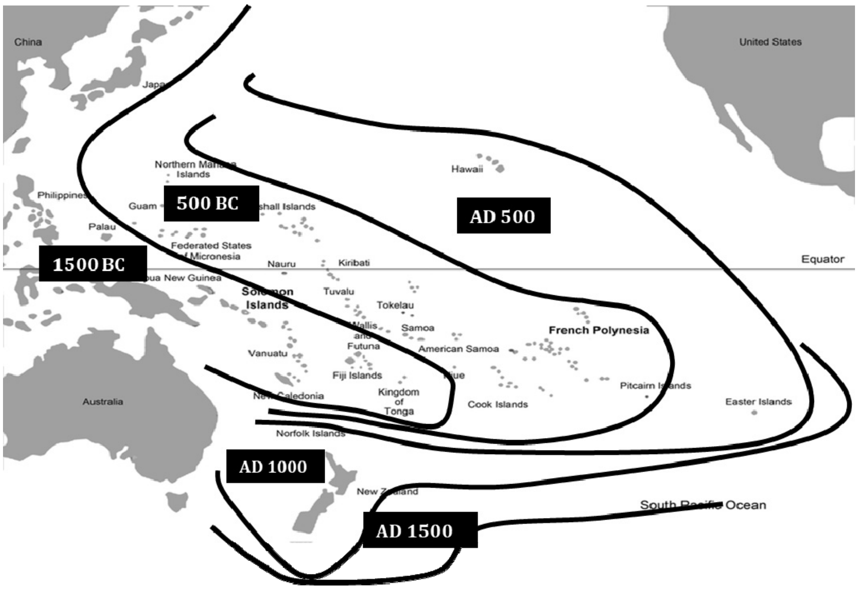


Fig. 9. Pulses in the colonisation of the Pacific

Source: Irwin 1992.

1.2. Complex Discovery Processes

As we have seen (in Fig. 2), high prices for exotic goods in the European colonial trading system encouraged exploration, which increased availability and caused exploration to fall back. However, another outcome was possible. When increased competition caused revenues to fall, entrepreneurs could respond with innovations that restored their revenues, for example through reducing costs, differentiating their products, or reaching markets more quickly (Steensgaard 1990: 151–152). For example, tobacco farmers responded to falling prices by packing bales more tightly, which reduced shipment charges, or they switched production to other crops. Similarly, the innovations in the British textile industry that drove the Industrial Revolution were stimulated by competition from Indian imports.

Let us call this a *complex discovery process* (see Fig. 10). It is a discovery process because there is innovation. It involves two kinds of negative feedback. Increased revenue causes competition that decreases revenue, and decreased revenue causes innovation that increases revenue. Note that innovation moves in the opposite direction to revenue so the influence of revenue on innovation is shown as negative in the figure.

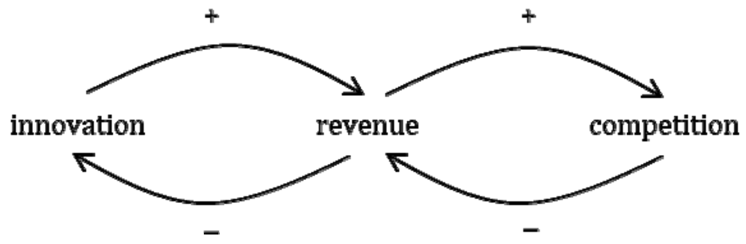


Fig. 10. A complex discovery process

There were perhaps other negative feedbacks besides these. For example, the more countries exploited their trading monopolies, squeezing profits out of them, the more they encouraged rivals to break those monopolies by means of piracy, military force, theft of trade secrets or substitution with similar goods (Steensgaard 1990: 123, 130). While Fig. 10 represents only one type of complex discovery process, it is the one that will be considered here, and it turns out to be capable of reproducing the structure of the Kondratieff cycle.

2. Mathematical Principles

2.1. Simple Harmonic Oscillator

Systems that involve both lag and negative feedback will tend to oscillate, that is exhibit cycles. The negative feedback ensures that the system does not keep changing in one direction but tends to return towards some central position. The lag ensures that it responds too late, overshooting the central position and thus oscillating around it rather than converging on it.

The simplest example of such a system is the simple harmonic oscillator, which is described by the differential equation (see Eq. 1). The variable x will oscillate around the origin. ω is a constant and t is time.

$$\frac{d^2x}{dt^2} = -\omega^2 x. \qquad \text{(Eq. 1)}$$

The solution to Eq. 1 has the form

$$x = a\sin(\omega t + \phi), \qquad \text{(Eq. 2)}$$

where a, the amplitude, and ϕ, the phase, are constants that arise in the course of the solution and whose values will depend on the initial conditions. ω is revealed to be the angular frequency, which means that it is the amount of angle moved per second. Since there are 2π radians (360 degrees) in a complete cycle, ω is 2π times the number of cycles per second.

The solution (Eq. 2) is illustrated in Eq. 11. The period of the cycle is $2\pi/\omega$, while the phase ϕ determines the position of the cycle relative to the start time, $t = 0$.

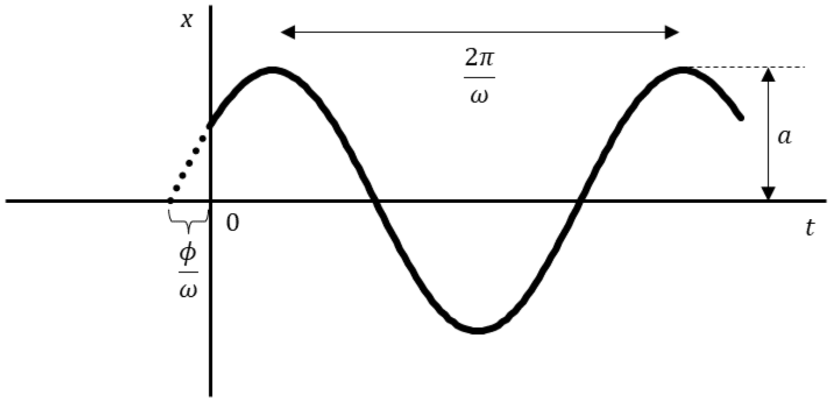


Fig. 11. Sinusoidal oscillation of simple harmonic oscillator

If we define the zero of time as the moment when the cycle is passing through the central point in a positive direction, then the distance ϕ/ω in Fig. 11 will be zero. Hence, the phase ϕ will be zero and we can ignore it. This simplifies the equations and does not change the basic shape of the cycle, which is what we are interested in. The adoption of this simplification below will be signalled with the words such as 'where we ignore the phase'.

Eq. 2 applies when the oscillations occur around $x = 0$, which means that x is as often negative as positive. If the oscillations occur not around zero but around a constant level k, we need to rewrite the right hand side of the equation as follows:

$$\frac{d^2x}{dt^2} = \omega^2 (k - x). \qquad \text{(Eq. 3)}$$

The solution is

$$x = k + a\sin(\omega t), \qquad \text{(Eq. 4)}$$

where we ignore the phase, and the oscillation is as depicted in Fig. 12. If $k \geq a$, the quantity x will always be positive.

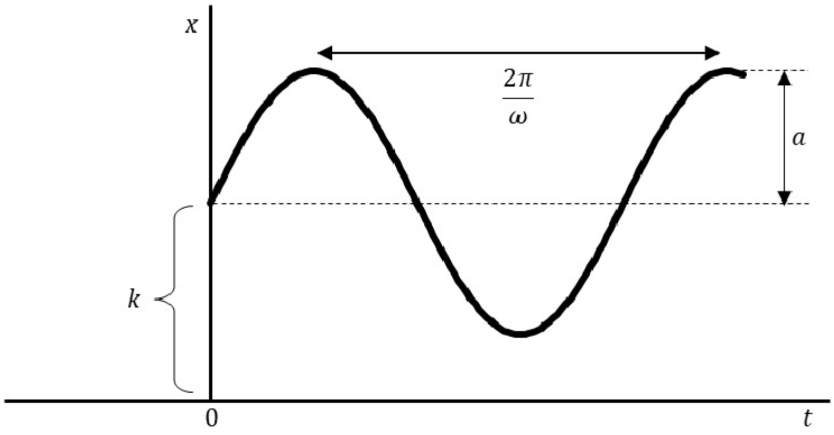


Fig. 12. Sinusoidal oscillation around a constant level

2.2. Systems Producing Cycles

An equation of the form of Eq. 1 can arise when we have two variables, x and y, that are related by a pair of equations of the form

$$\frac{dx}{dt} = \lambda y \qquad \text{(Eq. 5)}$$

and

$$\frac{dy}{dt} = -\mu x. \qquad \text{(Eq. 6)}$$

One can represent these equations diagrammatically as in Eq. 13. y causes x to increase (Eq. 5) and x causes y to decrease (Eq. 6).

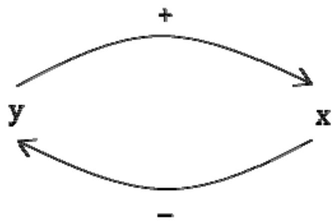


Fig. 13. Interaction of x and y implied by Eqs 5 and 6

If we differentiate Eq. 5 and substitute from Eq. 6, we get

$$\frac{d^2x}{dt^2} = \lambda \frac{dy}{dt} = -\lambda\mu x. \qquad \text{(Eq. 7)}$$

This is equivalent to Eq. 1 with $\omega^2 = \lambda\mu$ or $\omega = \sqrt{\lambda\mu}$. We can also differentiate Eq. 6 and substitute from Eq. 5 to get

$$\frac{d^2 y}{dt^2} = -\mu \frac{dx}{dt} = -\lambda\mu\, y. \qquad \text{(Eq. 8)}$$

Hence, y oscillates like x and with the same value of ω, thus with the same period (= $2\pi/\omega$).

Ignoring phase, we know that the solution to Eq. 1 is given by Eq. 2, so in this case the solution to Eq. 7 is

$$x = a\sin\left(\sqrt{\lambda\mu}\, t\right). \qquad \text{(Eq. 9)}$$

While we can move the x cycle freely in time to get rid of the phase, that is to set its starting phase to zero so that we can ignore it, one cannot do the same for y without destroying the temporal relationship (phase relationship) between y and x. Instead, we need to move y by the same amount as we move x. In other words, we can ignore the absolute phase, but one should not ignore the relative phase (the phase difference) between x and y, since this is important to the dynamics of the system described by these two variables.

To get a formula for y, one can use Eq. 5. Rearranging it to solve for y, and making use of Eq. 9, we get

$$y = \frac{1}{\lambda}\frac{dx}{dt} = \frac{1}{\lambda}\frac{d}{dt}\left(a\sin\left(\sqrt{\lambda\mu}\, t\right)\right) = a\sqrt{\frac{\mu}{\lambda}}\cos\left(\sqrt{\lambda\mu}\, t\right), \qquad \text{(Eq. 10)}$$

where the last step uses the fact that $d(\sin(\omega t))/dt = \omega \cos(\omega t)$ along with some minor algebraic manipulation. It can be checked that, if we substitute this formula for y into Eq. 6, we get back Eq. 5, so everything is consistent.

The two key points here are that:

1. x and y have the same frequency, $\sqrt{\lambda\mu}$, and therefore the same period, $2\pi / \sqrt{\lambda\mu}$. This means that they maintain a constant phase difference.

2. If x varies like sin ωt, then y varies like cos ωt. Since, by the laws of trigonometry, $\cos\theta = \sin(\theta + \pi/2)$, one can say that y varies like $\sin(\omega t + \pi/2)$. Hence, the phase difference between x and y is $\pi/2$, or a quarter of a cycle, with y leading x (or x lagging y).

The relationship of x and y is shown in Fig. 14.

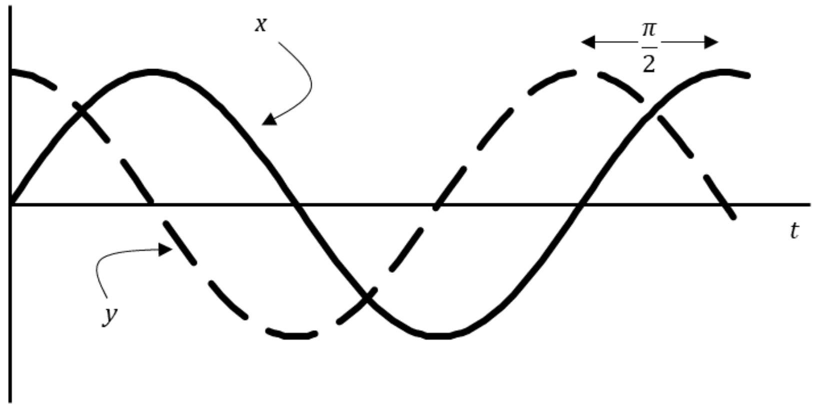


Fig. 14. Relationship of cycles in *x* and *y* arising from Eqs 5 and 6

2.3. Link between Cycles and Logistics

A logistic curve, of the kind indicated by the fitted line in Fig. 4, arises as the solution to an equation of the form

$$\frac{dx}{dt} = r\,x\left(1 - \frac{x}{x_{max}}\right), \qquad \text{(Eq. 11)}$$

where r and $x_{\max}$ are constants such that r controls the growth rate and $x_{\max}$ is the limiting (maximum possible) value of x. The solution looks like the following:

$$x = \frac{x_{max}}{1 + e^{-r(t - t_{1/2})}}, \qquad \text{(Eq. 12)}$$

where $t_{1/2}$ is the time when the growth is half-complete.

If we differentiate this, we get

$$\frac{dx}{dt} = \frac{r\,x_{max}\,e^{-r(t - t_{1/2})}}{\left(1 + ke^{-r(t - t_{1/2})}\right)^2}. \qquad \text{(Eq. 13)}$$

Fig. 15 (b) shows the logistic curve followed by the variable *x*, and Fig. 15 (a) shows its derivative. Alternatively, one can say that the logistic curve, on the right, is the integral of the curve on the left. To give an intuitive grasp, if Fig. 15 (a) represents the number of new cases per day in a pandemic, then Fig. 15 (b) represents the cumulative number of cases.

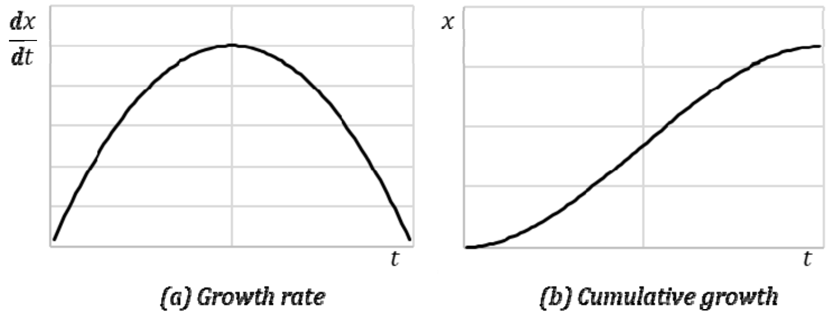


Fig. 15. A logistic curve (b) and its derivative (a)

While Fig. 15 shows a true logistic curve, any quantity that rises and falls from close to zero in a reasonably symmetric manner, that is similar to Fig. 15 (a), will produce a cumulative total that looks similar to Fig. 15 (b). Two such quasi-logistics are shown in Figs 16 and 17. The pulse that rises and falls is on the left and the cumulative curve is on the right. Notice that Marchetti's data in Fig. 4 deviates in a somewhat systematic way from the fitted logistic curve. This could be because the underlying dynamic is not truly logistic and corresponds to a quasi-logistic process like those of Fig. 16 or 17.

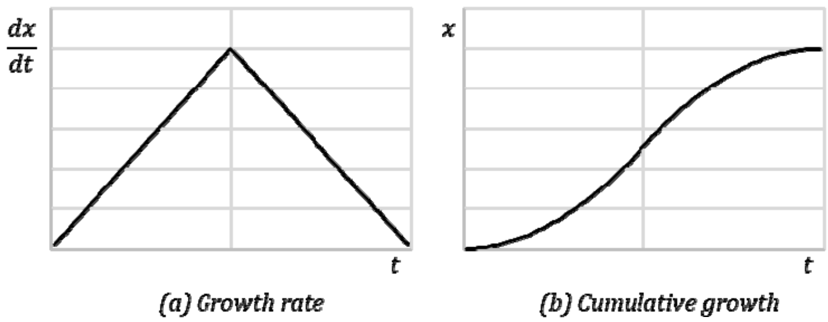


Fig. 16. Triangular pulse (a) and its quasi-logistic cumulative behaviour (b)

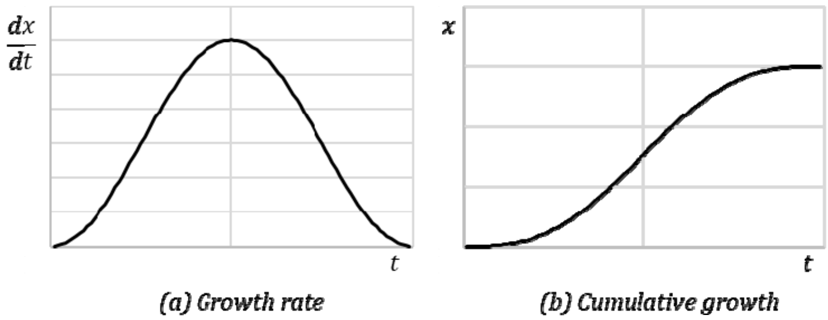


Fig. 17. Sinusoidal pulse (a) and its quasi-logistic cumulative behaviour (b)

In Fig. 17, the pulse that produces logistic behaviour is a part of a sine wave. An ordinary sine wave will not work since it oscillates around zero whereas the pulse must remain positive so that the cumulative total is always increasing. However, a sine wave can oscillate around a constant level, as in Fig. 12, and if this level is greater than 1, the value will always be positive (since the minimum value of sine is -1).

In the case of Fig. 17 (a), this is a part of a sine wave oscillating around 1. If the sine wave continues onward, giving a series of pulses, we will obtain a series of stacked logistics, as shown in Fig. 18.

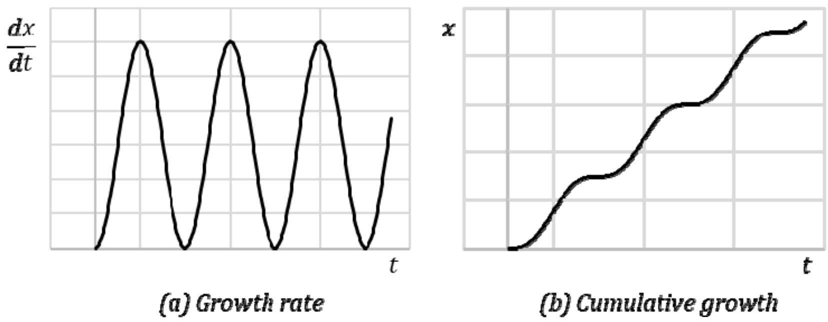


Fig. 18. A series of sinusoidal pulses (a) producing a series of stacked quasi-logistics as their cumulative total (b)

If the cycle oscillates around a level greater than 1, the cumulative total may still look somewhat logistic-like, though the curve will not flatten so much between pulses (see Fig. 19).

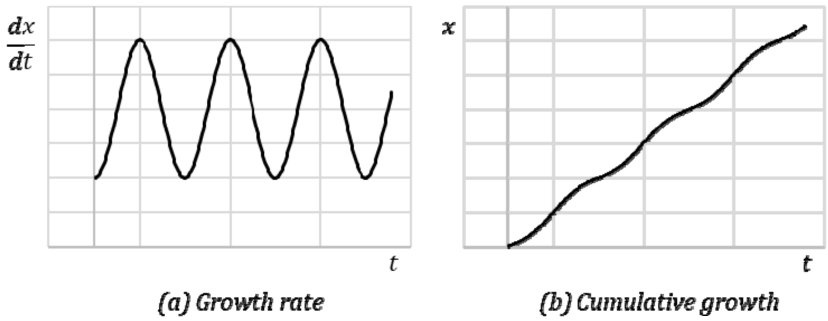


Fig. 19. Pulses that do not return to zero (a) and resulting quasi-logistic behaviour (b)

Thus, cycle-like processes that remain always positive will have logistic-like cumulative behaviour. Where we find cycles, we will also find logistics, and where there are logistics, there will also be cycles. The two phenomena are closely related.

The activities of interest in discovery processes, namely discoveries and innovations, will remain always positive because we do not make negative discoveries or take discoveries away (except perhaps in social collapses and dark ages). They also fluctuate cyclically, for reasons discussed in Section 0, and they exhibit successive logistics. We now know that these things are connected, with relatively straightforward mathematical principles behind them, as will be explored in the next section. Note that, in Fig. 7, although the successive logistics are shown side by side, they are really stacked logistics because the explorations of each pulse are added to the explorations of the preceding pulses. The same is true of Marchetti's invention wave of Fig. 4. It is one segment of a stacked logistic, since this wave built on the inventions of previous waves and was in turn built upon by subsequent waves.

3. Analysis

3.1. Simple Discovery Processes

If we compare the simple discovery processes of Fig. 1, Fig. 2, and Fig. 5 with the depiction of a simple harmonic oscillator in Fig. 13, we can see that simple discovery processes may be understood as simple harmonic oscillators and will be described by equations like those of Eqs 5 and 6. In this section, we will analyse the exploration-exploitation dynamic of Fig. 1. The others do not need separate analysis since they are structurally equivalent to it.

We will use X to represent exploration and P to represent exploitation. We know that a change in X produces a change in P.

Let us suppose specifically that a given *fractional* (or percentage) change in exploration causes a corresponding *fractional* change in exploitation. For example, if exploration doubles, exploitation doubles; or if exploration increases by 30 %, exploitation also increases by 30 %. This makes sense. Suppose that there is a given average number of river systems per mile of African coastline, and that each river system provided Portuguese entrepreneurs with the opportunity to set up a given average number of trading posts. Then doubling the exploration effort, in terms of miles of coastline explored, should on average double the number of river systems discovered and thus provide double the number of opportunities to set up trading posts for commercial exploitation.

The fractional changes in X and P can be written as dX/X and dP/P, respectively, where dX and dP are the absolute changes. Thus, we expect a change dX/X to produce a change dP/P, where

$$\frac{dP}{P} = \frac{dX}{X}. \qquad \text{(Eq. 14)}$$

However, we should not expect a change in exploration to produce an immediate change in exploitation. The economic system requires some time to respond to market signals. In terms of our example, news of the discoveries needs to get back to Portugal, then Portuguese entrepreneurs need to raise capital, hire crews, equip ships, travel back to where the opportunities are, develop commercial relationships with the locals, secure distribution deals for their products, and so on. The response time will likely be measured in years if not decades.

Let us use τ to designate the time required for a change in exploration to feed through the economy and produce a corresponding change in exploitation. We can now relate X and P through the equation

$$\text{rate of fractional change of exploitation} = \frac{\text{amount of fractional change of exploitation}}{\text{time taken to achieve change}} \qquad \text{(Eq. 15)}$$

By definition

$$\text{amount of fractional change of exploitation} = \frac{dP}{P} \qquad \text{(Eq. 16)}$$

and so

$$\text{rate of fractional change of exploitation} = \frac{d}{dt}\left(\frac{dP}{P}\right). \qquad \text{(Eq. 17)}$$

By Eq. 14, we also have

$$\text{amount of fractional change of exploitation} = \frac{dP}{P} = \frac{dX}{X} \qquad \text{(Eq. 18)}$$

and we have defined

$$\text{time taken to achieve change} = \tau. \tag{Eq. 19}$$

By substituting Eq. 17 on the left hand side and Eqs 18 and 19 on the right hand side, Eq. 15 becomes

$$\frac{d}{dt}\left(\frac{dP}{P}\right) = \frac{dX / X}{\tau}. \tag{Eq. 20}$$

We know, by elementary calculus, that

$$\frac{d}{dz}\left(\ln z\right) = \frac{1}{z} \tag{Eq. 21}$$

which can be rearranged as

$$d\left(\ln z\right) = \frac{dz}{z}. \tag{Eq. 22}$$

Thus, if we define $x = \ln X$ and $p = \ln P$, we can write dX/X as dx and dP/P as dp. This means we can rewrite Eq. 20 as

$$\frac{d}{dt}\left(dp\right) = \frac{dx}{\tau}. \tag{Eq. 23}$$

By a slight rearrangement of each side, this becomes

$$d\left(\frac{dp}{dt}\right) = d\left(\frac{x}{\tau}\right). \tag{Eq. 24}$$

Since the expressions within the $d(\ldots)$ on each side are equal (or equivalently, by integrating both sides), we obtain

$$\frac{dp}{dt} = \frac{x}{\tau}. \tag{Eq. 25}$$

In a similar way, by assuming that a given percentage change in exploitation produces an equal *but opposite* percentage change in exploration (because exploitation has a negative effect on exploration; see Fig. 1), and using σ to represent the time for the change to feed through the system, we obtain

$$\frac{dx}{dt} = -\frac{p}{\sigma}. \tag{Eq. 26}$$

Eqs 25 and 26 are the equivalents of Eqs 5 and 6. Given the discussion of Section 2.2, we know that these equations will generate sinusoidal oscillations for x and p, with x leading p by a quarter of a cycle. Comparison of the equations shows that λ corresponds to $1/\tau$ and μ corresponds to $1/\sigma$, so that $\omega = \sqrt{\lambda\mu}$ becomes $\omega = 1/\sqrt{\tau\sigma}$ and the period, given by $2\pi/\omega$, becomes $2\pi\sqrt{\tau\sigma}$. Writing T for the period, we have $T = 2\pi\sqrt{\tau\sigma} \Rightarrow \sqrt{\tau\sigma} = T/2\pi$. In the absence of other information, let us assume that the market response times of exploration to exploitation and of exploitation to exploration were the same, *i.e.* $\sigma = \tau$. Then we have $\sqrt{\tau\sigma} = \sqrt{\tau^2} = \tau$, so that $\tau = T/2\pi$. If we take the period of the exploration-exploitation cycle to be $T = 50$ years, corresponding to the interval between the inauguration of Portugal's exploration programme and its reinvigoration (see Section 1.1), then $\tau = 8$ years. If we take the period to be $T = 100$ years, as in Fig. 7, then $\tau = 16$ years. In principle, provided suitable data can be found, these values could be checked against the typical timescales to obtain backing, conduct voyages, *etc.* For present purposes, they are illustrative, demonstrating the concept of a simple discovery process, its application, and the possibility of quantitative verification.

By Eqs 25 and 26, x and p will oscillate about zero, being as often negative as positive. However, if $x = \ln X$ is negative, X is still positive, just less than one. Therefore, while x and p take on negative values, the underlying quantities, exploration X and exploitation P, will always be positive. This in turn means, according to the discussion of Section 2.3, that the cumulative amount of exploration should exhibit logistic-like behaviour.

Fig. 20 shows the sinusoidal variation in x and the corresponding variation in X. It may be seen that exploration (X) is always above zero.

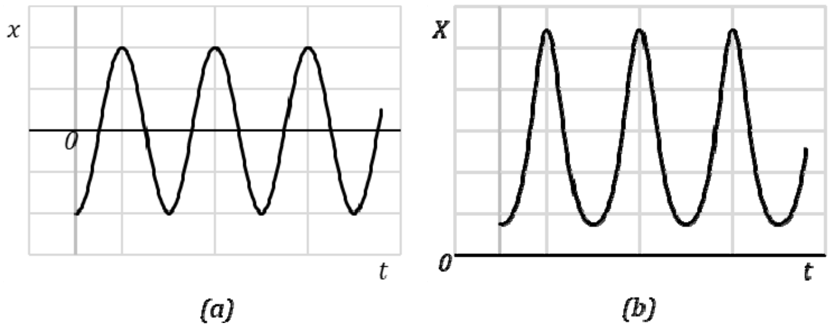


Fig. 20. Variation in (a) $x = \ln X$ and (b) X where X is exploration effort

Fig. 21 shows a single exploration pulse taken from Fig. 20 (b) along with the resulting cumulative amount of exploration. The logistic-like appearance is clear.

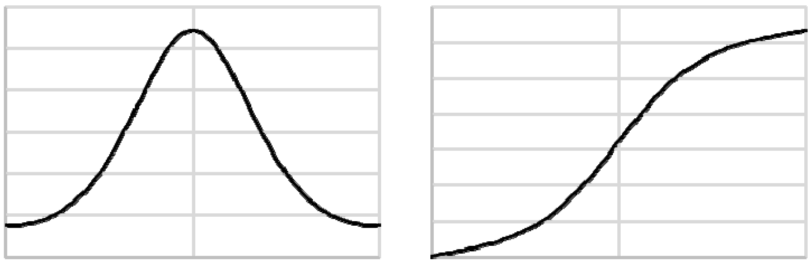


Fig. 21. Single pulse in exploration (a) and corresponding cumulative amount of exploration (b)

3.2. Complex Discovery Processes

Turning to complex discovery processes, we will use the same strategy as for simple ones by assuming that the relationships are between fractional changes in the underlying quantities and therefore can be expressed in terms of logarithms. Let *N*, *R* and *C* stand for innovation, revenue and competition, respectively, and let *n*, *r* and *c* stand for their respective logarithms.

The relationships shown in Fig. 10 are as follows:

• Innovation increases revenue and competition decreases it.

• Revenue increases competition.

• Revenue decreases innovation (this can be understood more intuitively as that a decrease in revenue causes an increase in innovation).

To turn these relationships into equations, we can again use the same strategy as for simple discovery processes, whereby the rate of change of the influenced variable depends on the characteristic timescale for a change in the influencing variable to propagate through the socio-economic system. Although, in principle, these timescales might be different for each variable, one can simplify the algebra by assuming that they are all the same, as we suggested in the case of simple discovery processes. At the end of this section, we will discuss why they might all be the same.

Choosing γ to represent the characteristic timescale for propagation of economic changes, the above points lead to the following equations:

$$\frac{dr}{dt} = \frac{1}{\gamma}(n - c) \qquad \text{(Eq. 27)}$$

$$\frac{dc}{dt} = \frac{1}{\gamma} r \qquad \text{(Eq. 28)}$$

$$\frac{dn}{dt} = -\frac{1}{\gamma} r. \qquad \text{(Eq. 29)}$$

Ignoring absolute phase, these equations can be solved to give:

$$r = \sqrt{2} a \sin(\omega t) \qquad \text{(Eq. 30)}$$

$$c = -a \cos(\omega t) + k \qquad \text{(Eq. 31)}$$

$$n = a \cos(\omega t) + k \qquad \text{(Eq. 32)}$$

where

$$\omega = \frac{\sqrt{2}}{\gamma} \qquad \text{(Eq. 33)}$$

and a and k are constants depending on the initial conditions. The validity of these solutions can be checked by differentiating each expression and showing they satisfy Eqs 27, 28 and 29. For instance, differentiating Eq. 32 gives

$$\frac{dn}{dt} = \frac{d}{dt}\left(a \cos(\omega t) + k\right) = -a\,\omega \sin(\omega t) = -a \frac{\sqrt{2}}{\gamma} \sin(\omega t) =$$
$$= -\frac{1}{\gamma}\left(\sqrt{2} a \sin(\omega t)\right) = -\frac{1}{\gamma} r$$

which is Eq. 29.

The typical solution is illustrated in Fig. 22. k is the constant level around which c and n oscillate, while a and $\sqrt{2}a$ are the amplitudes of the oscillations.

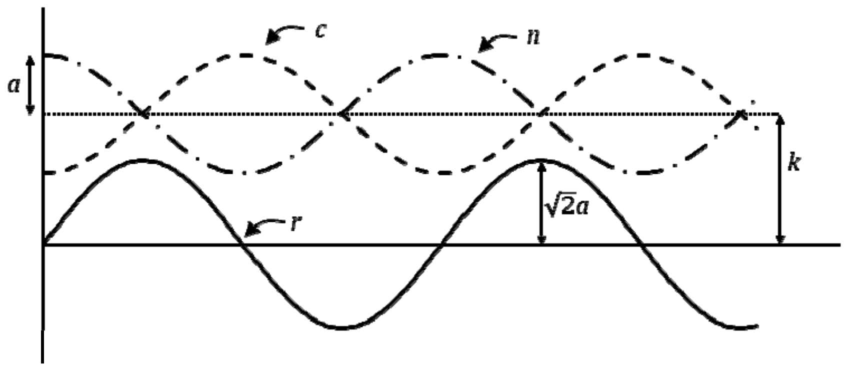


Fig. 22. Graphical representation of Eqs 30, 31 and 32

Using the fact that –cos x = sin(x – π/2) and cos x = sin(x + π/2), one can rewrite Eqs 30, 31 and 32 as follows:

$$r = \sqrt{2}a\sin(\omega t) \quad \text{(Eq. 34)}$$

$$c = a\sin(\omega t - \pi / 2) + k \quad \text{(Eq. 35)}$$

$$n = a\sin(\omega t + \pi / 2) + k \quad \text{(Eq. 36)}$$

The key points are therefore that:

1. The oscillations in revenue, competition and innovation all have the same frequency, so they maintain a constant phase relationship.

2. Competition lags revenue by a quarter of a cycle while innovation leads revenue by a quarter of a cycle.

The phase relationships of the three variables are illustrated in Fig. 23. The circle represents the cycle. Wherever on the cycle the revenue peak occurs, the competition peak will occur *π*/2 radians afterwards, and the innovation peak will occur *π*/2 radians before.

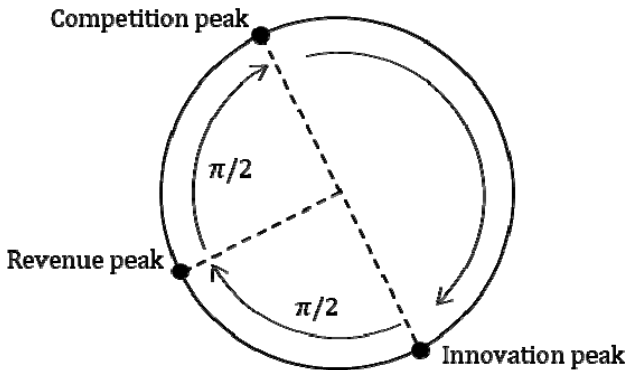


Fig. 23. Phase relationships resulting from Eqs 30, 31 and 32

We may compare this with Goldstein's empirical work on the Kondratieff cycle, which he calls the long cycle (Goldstein 1988: Ch. 12). Goldstein finds that the cycle links several politico-economic phenomena, as illustrated in Fig. 24. Innovation sets off a wave of economic activity, as for example the motor car or the smartphone organised economic activity around themselves, to manufacture, distribute and service the innovation. New technologies, combined with greater surplus due to the increase in economic production, give each country greater military potential, and this eventually results in wars, whose immediate causes originate in other geopolitical processes. The economic swings are accompanied by financial effects to do with investment, price level and real wages. Fig. 24 depicts the relative timings of the oscillations in each variable. The figure is based on Goldstein's own diagram and uses his nominal figure of 50 years for the cycle duration.

Fig. 24. Empirically discovered structure of the Kondratieff cycle
Source: Goldstein 1988: 259.

One can equate Goldstein's 'production peak' with the 'peak revenue' of our present model, insofar as both imply a peak of economic output. Similarly, we can equate his 'war peak' with the 'peak competition' of our present model, insofar as competition generates the friction and conflict that are likely to end in war (at least, the component of war associated with the Kondratieff cycle). Innovation has the same role in both models.

Fig. 25 is a repeat of Fig. 24 with highlighting of innovation, production and war along with their phase relationship. This is to facilitate comparison with Fig. 23. It may be seen that our present model generates the same relative structure as the corresponding parts of Goldstein's model, and thus provides a theoretical underpinning for his empirical results.

Fig. 25. A repeat of Fig. with highlighting of key variables and phase relationships

To understand the duration of the cycle, one can take inspiration from Devezas and Corredine (2001) and their suggestion that the Kondratieff cycle is connected to the turnover of generations. Thus, it is unlikely that people who already have occupations as say farmers, carpenters or domestic servants will restart their careers and decide to become sailors. If there is a demand for sailors, it will tend to be met by a new generation entering the workforce, seeing that employment opportunities for sailors are good and going into that career at the outset. In general, changes propagate through the socio-economic system not so much by existing economic actors switching their activity as by new actors that is youths, entering the system and gravitating towards whatever activity is a suitable response to the changed opportunities or threats. The timescale is thus related to the human ageing process and to the human generation time, which is the time required for economic actors to replace themselves. This timescale can be considered to apply to all kinds of socio-economic adjustment. This is why it was suggested we could assume a common timescale for all the interactions implied by Fig. 10, so that we adopted the same factor γ in Eqs 27, 28 and 29.

Let T_K and T_G represent the duration of the Kondratieff cycle and of a human generation respectively. Let us further suppose that a socio-economic change is achieved once the generation that is responding to the changed opportunity / threat has become the majority that is once half the old generation has been replaced by the new generation. The time for half a generation to be replaced is $T_G/2$. One can therefore write:

$$\gamma = \frac{T_G}{2}. \quad \text{(Eq. 37)}$$

The period of the cycle is given by $2\pi/\omega$, so that, by Eqs 33 and 37, we have

$$T_K = \frac{2\pi}{\sqrt{2}/\gamma} = \sqrt{2}\pi\gamma = \sqrt{2}\pi\frac{T_G}{2} = \frac{\pi}{\sqrt{2}}T_G \approx 2.2T_G. \quad \text{(Eq. 38)}$$

Assuming a generation time of 25 years, this gives $T_K \approx 55$ years. This is the typical duration of a Kondratieff cycle given by Devezas and Corredine (2001). Other work, using spectral analysis of economic data, suggests that the duration has been closer to 50 years and may have decreased to 45 years (Korotayev and Tsirel 2010). These smaller values for the Kondratieff period would imply a shorter generation turnover time of 20 to 22 years.

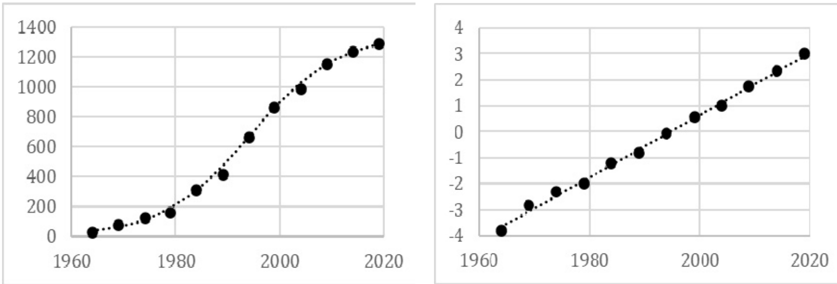
4. An Application to Space Colonisation

Jeff Bezos of Amazon has declared the ambition, through his Blue Origin company*, to have millions of people living and working in space. If we regard

* URL: https://www.blueorigin.com/.

space colonisation as a discovery process, one can track the progress of this objective by using the idea that it will follow a logistic '50-year pulsation'.

Fig. 26 plots the growth of the number of people who have been in space since the 1960s, showing that it fits a logistic curve to a high degree of accuracy. The process, which began around 1965 and seems to have been coming to an end around 2020, has lasted about 55 years. (The first 5 years of manned space travel was sporadic and experimental, and does not seem to have been part of the main logistic pulse.)



(a) Raw data *(b)* log(*f*/(1 – *f*))

Fig. 26. Growth of the number of people who have been in space, 1965–2020. In plot (b), *f* is the fraction of the limiting value achieved so far. Goodness of fit to a logistic trajectory, R^2 = = 0.9966

Source: Roberts 2020.

If the space exploitation pulse of the last 55 years is regarded as a complex discovery process, the point of maximum activity, in the middle of the pulse (*i.e.,* around 1990–1995) would correspond to the 'revenue' or 'production' peak (see Figs 23 and 25). That roughly this period was a time of maximum revenue in the space industry is suggested by Fig. 27, which shows the share price of Aerojet Rocketdyne, a key space stock, relative to the overall stock market. (The use of a relative price factors out general market fluctuations, such as the crash of 1987.) The perceived value of the stock peaked a little before 1990.

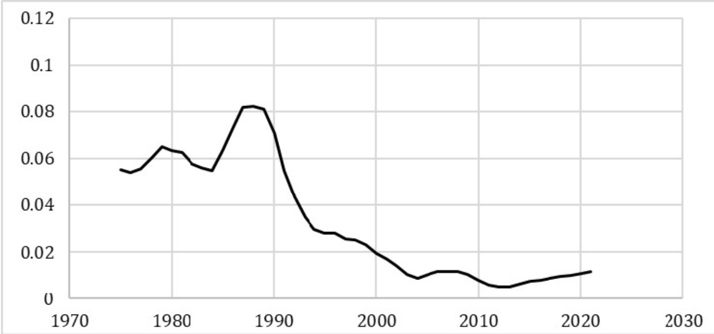


Fig. 27. Decadal moving average share price of Aerojet Rocketdyne relative to Standard & Poor 500 stock index

Source: Wolfram|Alpha, query 'Aerojet Rocketdyne'.

By Fig. 23, one should also expect competition to peak a quarter of a cycle after the revenue peak, which in this case would be around 2004–2008. A possible indication of this is given in Fig. 28, which shows that the founding of national space agencies peaked around this time.

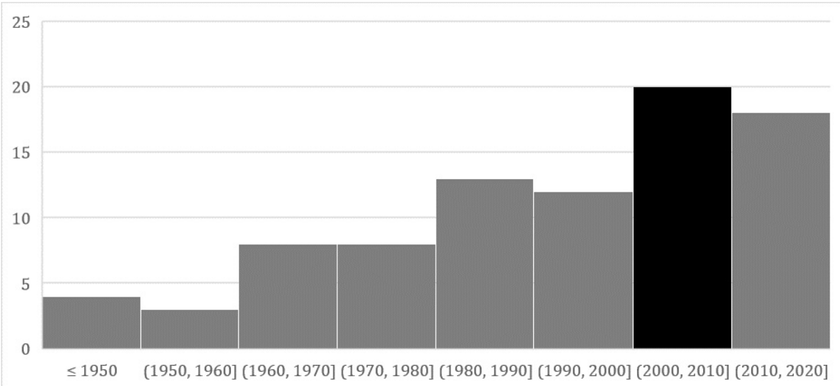


Fig. 28. Number of national space agencies founded by decade (2000s highlighted)

Source: UN 2021.

With the first logistic pulse of humanity's move into space having reached its completion around 2020, it follows that a new logistic pulse should now be beginning. To estimate the way in which the human presence in space will grow over the course of this next pulse, let us assume first that it will again last 55 years, and that it began in 2020. This means it should be concluded by 2075, which is consistent with the socio-technical periodisation proposed by Grinin,

Grinin, and Korotayev (2020). Let us further assume that the number of people in space at the end of the pulse (*i.e.,* in 2075), will be 10 million, which represents Bezos's 'millions of people' to the nearest order of magnitude. This number, referring to the instantaneous number of people in space, is somewhat different from the cumulative number of people in space, as plotted in Fig. 26. By making some assumptions about how long people stay in space, we could convert from the cumulative number of people who have been to space to the number of people actually in space in a given year. However, for an approximate estimate, let us assume directly that the number of people in space itself follows a logistic curve over the coming 55 years.

The space colonisation process will therefore be described by Eq. 12. We have chosen 10 million as the limiting value, so $x_{max} = 10^7$. If the start is in 2020 and the completion is in 2075, the midpoint will be 2047.5, therefore$t_{1/2}$ = 2047.5. In 2020, there were a mean number of 4.3 people in orbit (see Appendix 1, Table 2). Thus, we have $x = 4.3$, when $t = 2020$. Substituting all these values into Eq. 12, we obtain

$$4.3 = \frac{10^7}{1 + e^{-r(2020-2047.5)}} \qquad \text{(Eq. 39)}$$

which we can solve for r, to find $r = 0.53307$.

We can now use Eq. 12 to find x, the number of people in space, in each year t. The results are given for select years in Table 1. Note that the limiting value of 10 million is already achieved (to the nearest million) by 2060, which is 15 years before the nominal end of the cycle. Thus, there is exponential but, in retrospect, quite small growth between now and 2040, then a rapid growth between 2040 and 2060, and finally a plateauing with numbers stabilising close to the limiting value. The next round of space colonisation, perhaps leading to billions of people spreading through the solar system, would then begin around the 2080s.

Table 1. Growth of the number of people in space (round numbers), assuming a 55-year logistic growth pulse from 2020, ending with 10 million in 2075

Year	Number
2020	4
2021	7
2022	12
2025	62
2030	890
2035	13,000
2040	180,000
2048	5.7 million
2060 and thereafter	10 million

Table 1 suggests that we should have expected around seven people to be in space in 2021. In fact, the mean number of people in space over that year was 9.2 (see Appendix 1, Table 3). In these early years of the cycle, the precise numbers have a high margin of error, and one can conclude that the experience of 2021 was by no means incompatible with the projections of Table 1. 2021 was also notable for a number of achievements, such as the first fully private mission to the International Space Station, the first fiction film to be filmed in orbit, the largest number of people in orbit at one time, the oldest and youngest people officially to travel beyond the Kármán Line (the 'edge of space'), and the first occupation of China's Tiangong space station. Such a proliferation of 'firsts' is consistent with a process that is at the beginning of an expansion phase.

5. Conclusion

This paper has shown how economic cycles can be modelled in terms of 'discovery processes', which involve oscillations between an innovation / exploration phase and a production / exploitation phase. In the case of the Kondratieff cycle, where the economic dynamic is accompanied by a political dynamic, the model can incorporate an additional interaction involving feedback between competition and production / exploitation, and this further drives the oscillation. This represents the beginning rather than the completion of a research programme.

It has been assumed that the fundamental relationships are between fractional changes in the quantities of interest, so that a given fractional change in one quantity produces a corresponding fractional change in another quantity. This results in equations in which the variables are logarithms of the relevant quantities, and it means that, while the variables can take on negative values over the course of the cycle, the underlying quantities are always positive. This in turn means that their cumulative totals (*e.g.,* number of inventions) follow a logistic-like curve, which matches empirical findings.

The formalism used for the analysis has been the simple harmonic oscillator, whose behaviour is straightforward to understand. It is unlikely that the regular sinusoidal oscillations predicted by such a model are applicable in the real world, where the duration, amplitude and waveform of the oscillation typically vary from one cycle to the next. The value of the simple model is to improve intuition about the socio-economic processes that result in cycles. To some extent deviations of real data from the model's idealised behaviour may be explained in terms of random noise or the admixture of other processes. However, it may also be that the model's match to data could be improved by basing it on other mechanisms that involve lags and negative feedbacks and thus generate oscillations. For example, the relationships between the variables might be expressed better as a set of Lotka-Volterra equations, which have been used to model predator-prey cycles.

By adopting the insight of other researchers, the duration of the oscillation has been linked to the human generation time. It was suggested that behavioural change is achieved more readily by the influencing, for example through market forces, of uncommitted youngsters as they enter the socio-economic world than by the conversion of those already enmeshed in existing socio-economic networks. Change is thus controlled by the ageing process and by the resulting turnover of populations. In this respect, the specific analysis used here adopted some simple and stylised assumptions. It is an area where further investigation of the propagation of socio-economic adjustments and of the speed at which generations replace each other could improve the model's accuracy and predictive potential.

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Appendix 1

Number of Persons in Orbit (2020–2021)

This appendix shows the calculation of the mean number of persons in orbit in 2020 and 2021.

Table 2. Persons in orbit during 2020. An arrive date of 01/01/2020 indicates someone already in orbit at the beginning of the year, while a depart date of 31/12/2020 indicates someone who was still in orbit at the end of the year

Name	Arrival	Departure	Days
Koch Christina Hammock	01/01/2020	06/02/2020	36
Skvortsov Aleksandr Aleksandrovich Jr.	01/01/2020	06/02/2020	36
Parmitano Luca Salvo	01/01/2020	06/02/2020	36
Morgan Andrew Richard	01/01/2020	17/04/2020	107
Skripochka Oleg Ivanovich	01/01/2020	17/04/2020	107
Meir Jessica Ulrika	01/01/2020	17/04/2020	107
Ivanishin Anatoli Alekseyevich	09/04/2020	22/10/2020	196
Vagner Ivan Viktorovich	09/04/2020	22/10/2020	196
Cassidy Christopher John	09/04/2020	22/10/2020	196
Behnken Robert Louis	30/05/2020	02/08/2020	64
Hurley Douglas Gerald	30/05/2020	02/08/2020	64
Ryzhikov Sergei Nikolayevich	14/10/2020	31/12/2020	78
Kud-Sverchkov Sergei Vladimirovich	14/10/2020	31/12/2020	78
Rubins Kathleen Hallisey	14/10/2020	31/12/2020	78
Hopkins Michael Scott	16/11/2020	31/12/2020	45
Glover Victor Jerome	16/11/2020	31/12/2020	45
Noguchi Soichi	16/11/2020	31/12/2020	45
Walker Shannon	16/11/2020	31/12/2020	45
		TOTAL	1559
Mean number of persons in orbit over year (= TOTAL divided by 365 days)			4.3

Source: Spacefacts 2022.

Table 3. Persons in orbit during 2021. An arrive date of 01/01/2021 indicates someone already in orbit at the beginning of the year, while a depart date of 31/12/2021 indicates someone who was still in orbit at the end of the year. Note: 'in orbit' excludes those who entered space in brief, sub-orbital launches. ISS – International Space Station

Name	Facility	Arrive	Depart	Days
Ryzhikov Sergei Nikolayevich	ISS	14/10/2020	17/04/2021	185
Kud-Sverchkov Sergei Vladimi-rovich	ISS	14/10/2020	17/04/2021	185
Rubins Kathleen Hallisey	ISS	14/10/2020	17/04/2021	185
Hopkins Michael Scott	ISS	16/11/2020	02/05/2021	167
Glover Victor Jerome	ISS	16/11/2020	02/05/2021	167
Noguchi Soichi	ISS	16/11/2020	02/05/2021	167
Walker Shannon	ISS	16/11/2020	02/05/2021	167
Hopkins Michael Scott	ISS	16/11/2020	02/05/2021	167
Glover Victor Jerome	ISS	16/11/2020	02/05/2021	167
Novitsky Oleg Viktorovich	ISS	09/04/2021	17/10/2021	191
Kimbrough Robert Shane	ISS	23/04/2021	09/11/2021	200
McArthur Katherine Megan	ISS	23/04/2021	09/11/2021	200
Hoshide Akihiko	ISS	23/04/2021	09/11/2021	200
Pesquet Thomas Gautier	ISS	23/04/2021	09/11/2021	200
Dubrov Pyotr Valerievich	ISS	09/04/2021	31/12/2021	266
VandeHei Mark Thomas	ISS	09/04/2021	31/12/2021	266
Shkaplerov Anton Nikolayevich	ISS	05/10/2021	31/12/2021	87
Chari Raja Jon Vurputoor	ISS	11/11/2021	31/12/2021	50
Marshburn Thomas Henry	ISS	11/11/2021	31/12/2021	50
Maurer Matthias Josef	ISS	11/11/2021	31/12/2021	50
Barron Kayla Sax	ISS	11/11/2021	31/12/2021	50
Nie Haisheng	Tiangong	17/06/2021	17/09/2021	92
Liu Boming	Tiangong	17/06/2021	17/09/2021	92
Tang Hongbo	Tiangong	17/06/2021	17/09/2021	92
Zhai Zhigang	Tiangong	15/10/2021	31/12/2021	77
Wang Yaping	Tiangong	15/10/2021	31/12/2021	77
Ye Guangfu	Tiangong	15/10/2021	31/12/2021	77
			TOTAL	3874
Mean number of persons in orbit over year (= TOTAL divided by 365 days)				10.6

Source: Spacefacts 2022.